# Lab 5a: Magnetic Levitation (Week 1)

*"Magnetism, as you recall from physics class, is a powerful force that causes certain items to be attracted to refrigerators."*– Dave Barry

## 1 Objectives

The goal of this week's and the next week's lab is to design and implement an analog controller for the magnetic levitation (MagLev) system shown in Figure 1. To do so, we want to design an analog compensator such that the DC gain is 1000 A/m and the phase margin is 60 degrees.

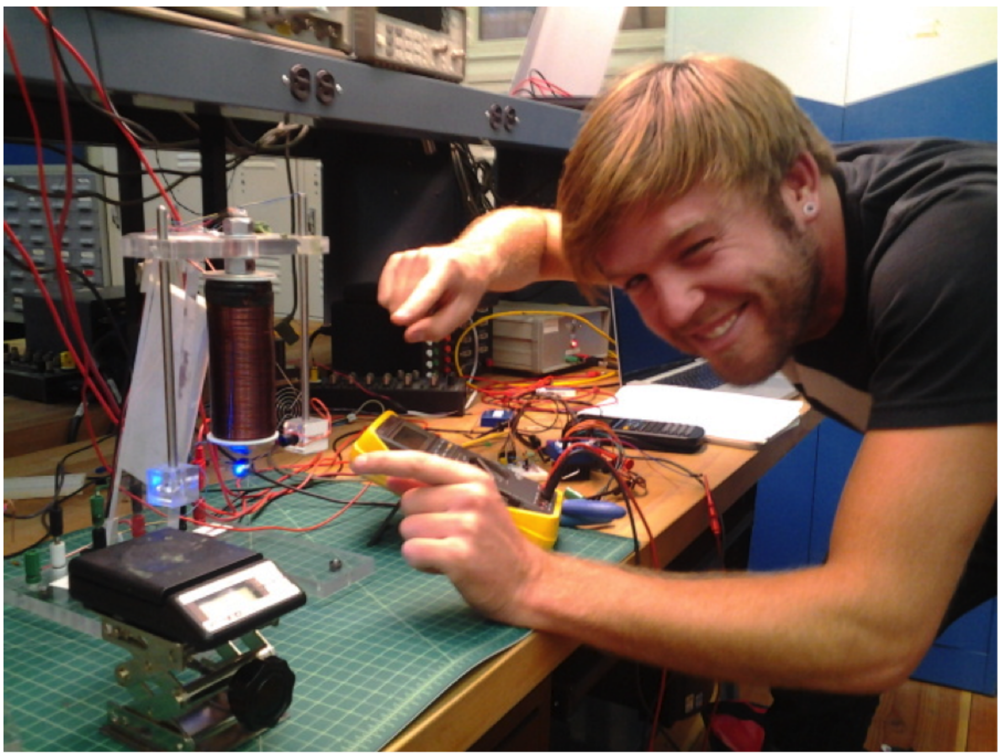

Figure 1: Magnetic levitation system in action (starring Skot Croshere)

## 2 Equipment and Safety

### 2.1 Lab Rules

- Safety glasses can be requested, if needed.
- The coils and black heat sink on the current amplifier can get very hot, please do not touch them.
- Before making any connections, make sure that the polarity is correct.
- Be cautious before turning on any power supplies and double check all connections.
- If you are unsure about something, ask first!

### 2.2 Lab Equipment

- Magnetic levitation system: magnetic coil, LED, and photoresistor
- Copley Controls Bantam BTM-055-20 current amplifier (Blue box mounted on green PCB)
- 24V DC power supply for current amplifier
- Various cables and wires
- $z$-stage
- Steel Balls
- Digital weighing scale

## 3 Theory

### 3.1 System Setup and Block Diagram

Figure 2 shows a high level block diagram of the magnetic levitation system. The main task of this first week of the lab is to perform system identification to obtain the transfer function of the linearized plant $G(s)$. Based on this model, you will design an analog controller to stabilize the system, using the methods you learned in class. In next week's lab you will then implement this controller using an analog circuit.

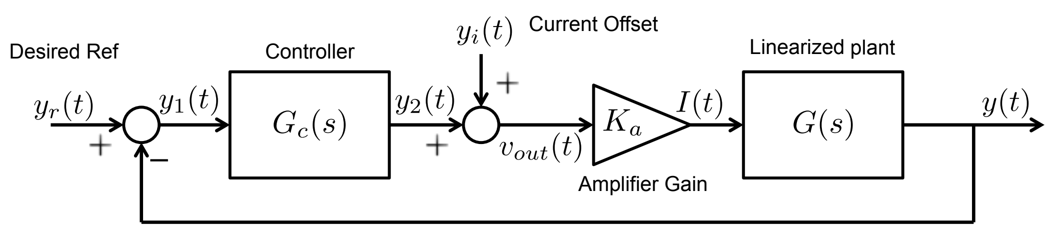


Figure 2: High level block diagram of the magnetic levitation setup

Figure 3 on the next page of these instructions shows a block diagram of the magnetic levitation setup with circuit level details. You will be picking values for $R_1$, $R_2$, and $C$ to stabilize the steel ball around an equilibrium point. The potentiometers will be used to calibrate the photoresistor and the current offset to the equilibrium point.

### 3.2 System Modeling

The equations of motion of the ball are:

$$m\ddot{x} = f(I, x) - mg \tag{1}$$
$$y = h(x) \tag{2}$$

where $x$ is the vertical position of the ball (in m), $I$ is the current through the coil (in A) and $g = 9.81\,\mathrm{m/s^2}$ is the gravitational constant. The *nonlinear* function $f(I, x)$ describes the magnetic force (in N) on the ball as a function of $x$ and $I$, and the *nonlinear* function $h(x)$ describes the voltage drop across the photo resistor as a function of $x$.

You will need to perform system identification on the MagLev plant in order to determine a linearized system model. This is unlike your previous labs where you were given the system parameters. You will notice that the most difficult part of this lab is in fact the system identification. But the point of the lab is to expose you to control systems design in the real world, so the effort is worth it!

# 4 Pre-Lab

## 4.1 Familiarization

Get an idea of the system setup by studying Figures 2 and 3 and the system modeling equations. A good understanding of the system will be very helpful when implementing the analog controller during the lab. Also brush up on linearization of nonlinear systems around an equilibrium point as well as op-amp and impedance theory.

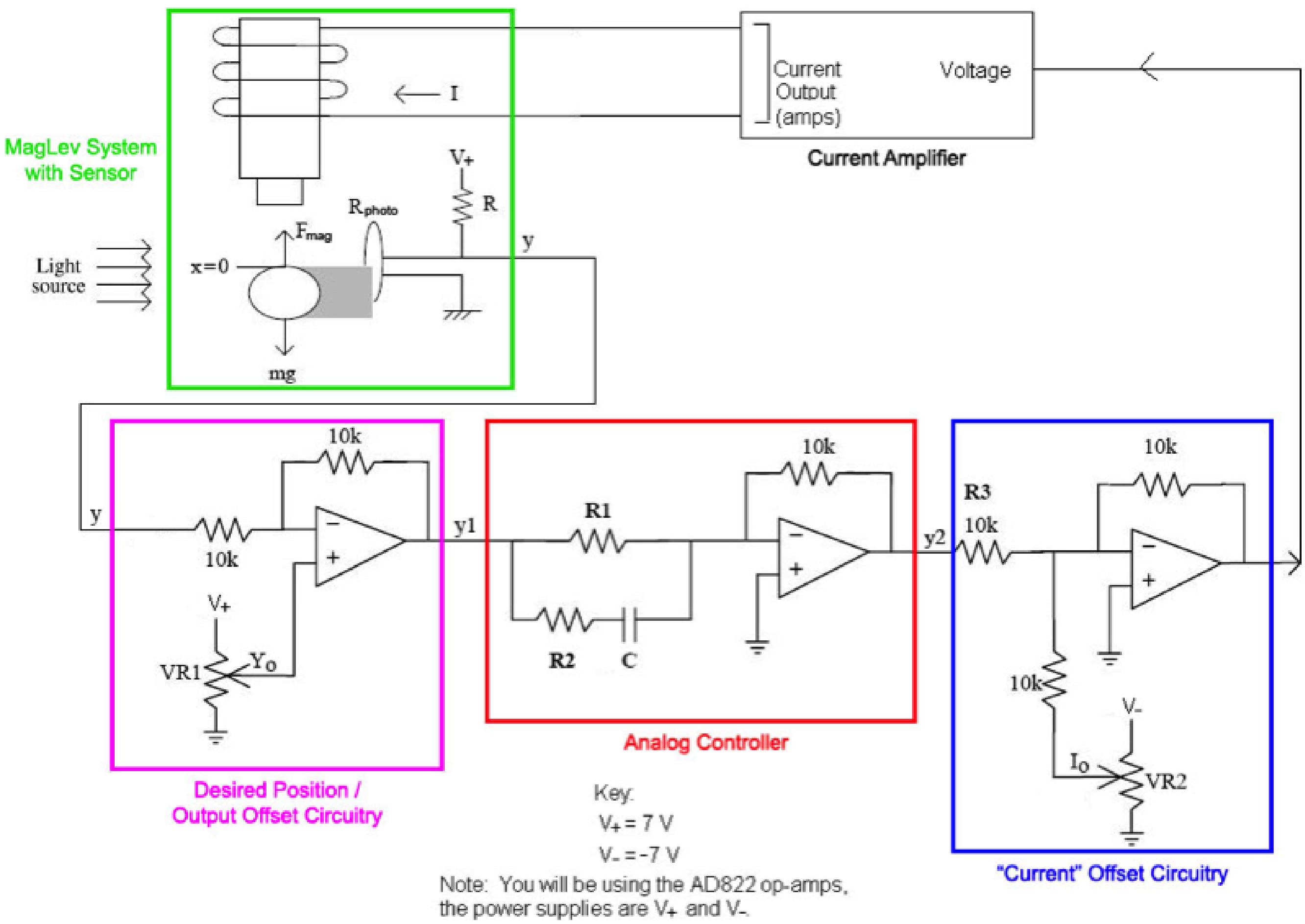


Figure 3: Block diagram of the magnetic levitation setup with circuit level details.

## 4.2 Derivation of the System Transfer Function

### 4.2.1 Desired position / output offset circuitry

Let $y_{ref} := Y_0$ in the left box in Figure 3. This is the reference voltage that you will adjust using a potentiometer. Derive the following relationship between the signal $y_1$, the voltage $y$ from the photoresistor and the reference voltage $y_{ref}$:

$$y_1 = 2y_{ref} - y \tag{3}$$

### 4.2.2 Transfer function of the analog controller

Derive the following transfer function of the analog controller in the red box in Figure 3:

$$G_c(s) = \frac{Y_2(s)}{Y_1(s)} = -\frac{10^4\,\Omega}{R_1} \cdot \frac{(R_1 + R_2)C \cdot s + 1}{R_2 C \cdot s + 1} \tag{4}$$

Notice the negative sign of this transfer function.

### 4.2.3 Current offset circuitry

In the current offset circuit in the blue box in Figure 3, let $y_i$ be the voltage drop across the potentiometer and let $V_{out}$ be the output of the op amp. Observe that $y_i$ will be negative. Derive the following relationship:

$$V_{out} = -(y_2 + y_i) \tag{5}$$

### 4.2.4 Linearization of the system

We linearize the equation of motion of the MagLev plant to obtain the following approximation:

$$m\ddot{x} = f(I, x) - mg \approx f(I_o, x_0) + K_i \delta I + K_x \delta x - mg \tag{6}$$

where $\delta I = I - I_0$ and $\delta x = x - x_0$. We will tune the current offset circuitry to cancel out the gravitational force at the equilibrium, i.e. we will have $f(I_0, x_0) = mg$. Also, we linearize the output function $h$ around the equilibrium $x_0$ to obtain $h(x) \approx a\delta x$, where $\delta x = x - x_0$. Therefore, our linearized plant is:

$$m\ddot{x} = K_i \delta I + K_x \delta x \tag{7}$$

$$y = a\delta x \tag{8}$$

Derive the following transfer function of the linearized plant:

$$G(s) = \frac{Y(s)}{I(s)} = \frac{aK_i}{m\left(s^2 - \frac{K_x}{m}\right)} \tag{9}$$

# 5 Lab

## 5.1 Setting up the Hardware

The hardware setups are all slightly different (mainly because of differences in the exact placement of the photoresistor). **Make sure you use the same hardware setup for both weeks of the lab** or you need to redo the system identification in Lab5a. Please do not make any changes to the setup (such as adjusting the photoresistor) without consulting the GSI first – you may be changing another group's setup.

You will not have to implement the complete circuit given in Figure 3 in this lab, that will happen next week. However, you will be using the current amplifier. The GSI will help you with setting things up. **Do not turn on your circuit before checking off with the GSI!** If you want to know more about the current amplified, you can find a data sheet under `http://www.copleycontrols.com/Motion/pdf/bantam.pdf`.

## 5.2 System Identification

In this lab, you will identify parameters of a linearized model of the magnetic levitation system. Make sure you are aware of the following:

- Remember to have proper SI units for all of these measurements and calculations and keep track of what you are measuring (i.e. force is in Newtons, not grams). Also, make sure your lighting conditions are consistent since you are using a photoresistor.
- A note on the power supplies: You may be using multiple power supplies in this lab. The low-power amplifier has a COM connection, which should be connected to the GND of **the +24V power supply**. For part 4 you will need to vary the voltage to the current amplifier. For this you can use the 6V channel on the lab power power supply. Note that this is quite sensitive, so be careful when adjusting.
- The high-power power supply +24V/GND will only be connected to the current amplifier board. If everything is ok on the current amplifier board, the AOK green LED will glow. If the LED turns off during operation it might be due to an automatic thermal shutoff, which should not occur if you are doing things correctly. Time to shut off the power supply and debug your circuit!
- The first offset circuits in Figure 3 is to scale $y$ down to $y_1$ such that $y_1$ moves around 0.

Perform system identification on the MagLev setup by following the steps below:

1. Decide on an equilibrium height of the ball, which is the $x = x_0 = 0$ position. This position should be set so that about half of the light going to the photo-resistor is covered (about 6 mm from the bottom of the electromagnet).
2. Measure the range of variation of the resistance of the photo-resistor as the balls shadow covers from none to the entire resistor surface. Using this data, determine a suitable resistor value $R$ to use in series with the photo resistor (see green system box in Figure 3), given that the supply voltage is $V_+ = 7V$ and the fact that the photo resistor must not exceed a power dissipation of 250mW. *Hint:* You can use the wooden piece to keep the ball from rolling around.

3. To linearize $h(x)$:
   (a) Position the ball at $x = 0$ (half shade on photo resistor) using the $z$-stage and weighing scale. Wire the photo resistor in series with a potentiometer and apply $V_+ = 7V$. Move the ball slightly up and down (within a few mm) from your equilibrium position. Record the output voltage $y$ at about 5 or 6 different positions.
   (b) Using Matlab, plot the output $y$ as a function of the deviation $\delta x$ from the equilibrium point $x = 0$. You should observe that this relationship is indeed nonlinear. Find the slope $a$ (in $V/m$) of a linear approximation of $h(x)$ (you can use different techniques for this, including determining the slope from a pair of measurements, or performing linear regression over a number of measurements – just make sure to get a good approximation *near the equilibrium point*). **What is the optimal equilibrium voltage of y?**
4. To determine $K_i$ (N/A):
   (a) Position the ball at $x = 0$ (half shade on photo resistor) using the $z$-stage and weighing scale.
   (b) To adjust the current through the coil, change the voltage to the current amplifier's "ref+" pin. You can determine the amplifier's output current from the voltage at the "current monitor" pin (to convert from voltage to current, multiply by the factor $\frac{20}{3} \cdot \frac{A}{V}$. Do not confuse this number with the amplifier gain $K_a$). Set the current so that the ball is almost weightless ($\approx 0.5\,\text{g}$) and record $I_0$. *Hint:* The magnetic field will have an effect on the reading of the scale. Make sure to tare the scale every time when the magnetic field is modified.
   (c) Decrease voltage to the current amplifier so that ball gets slightly "heavier". Record a few current/weight pairs. If you are careful you may also be able to slightly increase the voltage, so long as the ball does not lift off and stick to the electromagnet. Remember, the scale can never measure a "negative" weight. Any negative readings you get on the scale is not reasonable.
   (d) Using Matlab, plot the weight of the ball as a function of the deviation $\delta I$ from the equilibrium point $I_0$. How does this relationship compare to the voltage/displacement relationship from part 3b? In particular, how accurate is a linear approximation in either case? What do you think the reason for this is?
   (e) Compute $K_i$ from the recorded values. Again, make sure to get a good approximation *around the equilibrium point* $I_0$.
5. To determine $K_x$ (N/m):
   (a) Position the ball at $x = 0$ (half shade on photo resistor) using the $z$-stage and weighing scale.
   (b) As before, to adjust the current through the coil you adjust the voltage to the current amplifier "ref+" pin. Set the current so that the ball is weightless and record $I_0$. Make sure to tare the scale with the current on! (as the magnetic field influences scale reading).
   (c) Decrease the ball height in small steps and record the new height/weight pairs.
   (d) Plot the weight of the ball as a function of the deviation $\delta x$ from the equilibrium point $x = 0$. Discuss this relationship in comparison to the other two you have already determined.
   (e) Compute $K_x$ from the recorded values. Again, make sure to get a good approximation *around the equilibrium point* $x_0$.

6. Let us analyze the DC gain of your system (from $y$ to current out). The ball's position is first sensed, then passed through your controller, and then converted to a current that is sent through the coils of the electromagnet. So the DC gain is the sensor gain multiplied by the DC gain of your controller ($K_c$) multiplied by the gain of the current amplifier ($K_a$). In variables:

$$\text{DC gain} = a\,K_c\,K_a \tag{10}$$

   Now, we want the DC gain of our circuit to be 1000 A/m. Assume $K_a = 2\,\text{A/V}$ and then use this along with the value of $a$ determined in part 3b to calculate $K_c$.

7. If your group finished the above process early, familiarize the process a couple more times. (Try to change the direction of the LED a littie bit and you will be surprised how much it affect the parameter.) You might need to re-identify these parameters in the lab next week.

## 5.3 Building up the OpAmp Circuit

For the second part of this lab, you will build an op amp circuit so that you can get used to debug op amp circuits, which you will build in the next week. For this lab, you need to build "Desired Position/Output Offset Circuitry" in Figure 3. Since we do not connect the photoresistor to the circuit this week, for the voltage $y$, please use the power supply.

1. Build "Desired Position/Output Offset Circuitry" in Figure 3 (For the op amp, please look at Figure 4)
2. Supply any voltage combination (less than 4V) for $y_{ref}$ and $y$ and measure/record the corresponding voltage $y_1$ (Repeat this process for at least 5 different combinations of voltage $y_{ref}$ and $y$)
3. Using an equation 3 from Prelab, calculate the voltage $y_1$
4. Compare your recorded and calculated voltages of $y_1$ and check if they are similar

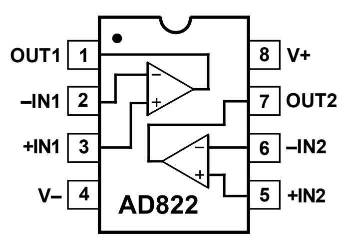


Figure 4: Connection Diagram AD822 OpAmp.

## 5.4 Lab Report

Your group will hand in a mini report for lab 5a and a full-length report for lab 5b. In your report for this lab, describe the linearization method you used and how you found the constants for the model. Be sure to write out the full linearized equations with the values for your identified system parameters. Also, write down all 5 different combinations of $y_{ref}, y$, recorded and calculated $y_1$ in your mini report. The report should only be a paragraph or two (and 1 table for the Lab 5.3).